Learning to learn by Gradient descent by Gradient descent

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Introduction to Optimisation Problems And Gradient descent

- Machine Learning ⇒ Optimization of some function f:

$$Find\ \theta^* \in argmin_{(\theta \in \Theta)} f(\theta)$$

- Most popular method: Gradient descent (Hand-designed learning rate)

$$\theta_{t+1} = \theta_t - \alpha \nabla(\theta_t)$$

- Better methods for some particular subclasses of problems available, but this works well enough for general problems

The Key Concept

(Optimiser and Optimisee)

- Use **learned** update rule instead:

$$\theta_{t+1} = \theta_t + g_t(\nabla f(\theta_t)), \phi)$$

- g is the *optimiser* function parameterized by ϕ
- It is implemented using a RNN which maintains its own state h_t
- It outputs the update rule g_t to be used for the optimisee function f

Loss Function

- The loss function for g is defined as:

$$L(\phi) = E_f[\sum_{t=1}^{T} \omega_t f(\theta_t)]$$

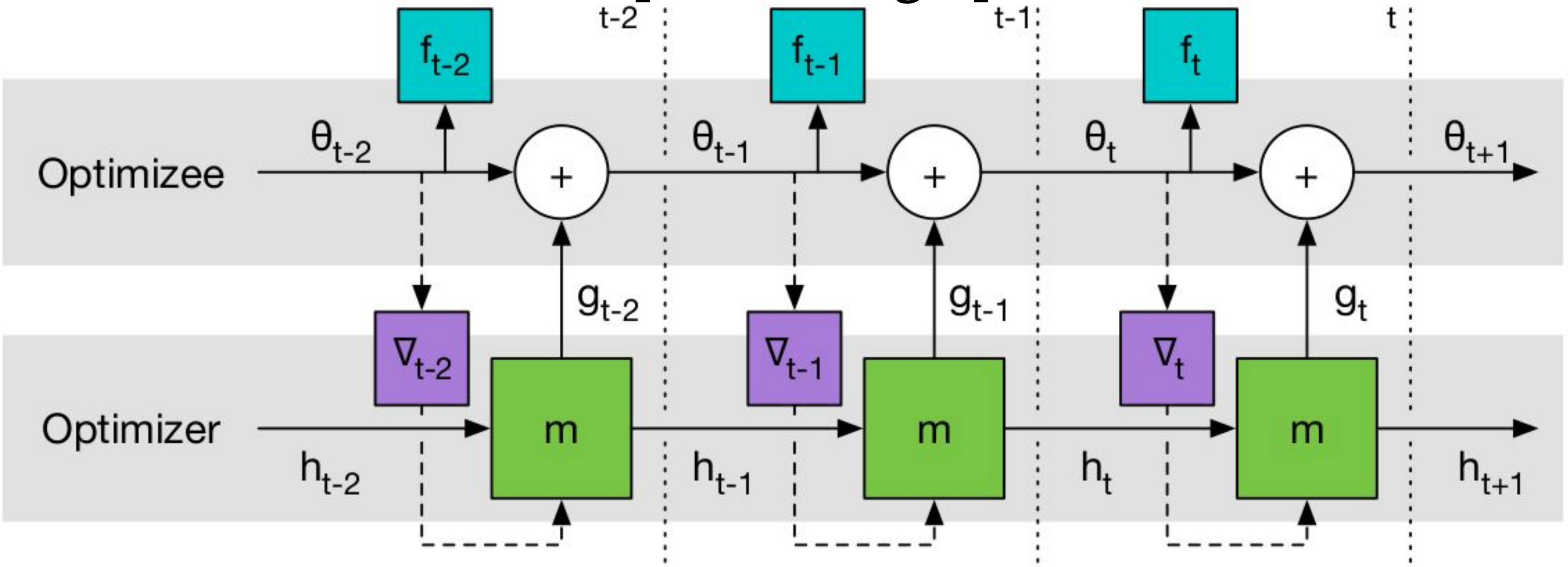
$$\theta_{t+1} = \theta_t + g_t;$$

$$where \begin{bmatrix} g_t \\ h_{t+1} \end{bmatrix} = m(\nabla_t, h_t, \phi)$$

- For this loss function, we train the optimiser for different datasets
- This is done using gradient descent

(Hence the title)

Computational graph



Assuming $\delta\nabla_t/\delta\phi = 0$, i.e. no need to compute second or higher derivatives of f. Hence gradients along the dashed lines neglected.

What we did

- Implemented optimiser function using LSTM (Long Short Term Memory) Architecture using the PyTorch library to utilise the .backward() function to conveniently calculate the gradients to be used in meta optimizer.

- Compared this with industry standard hand-designed techniques like ADAM, RMSprop, to do the same as the meta optimiser and compared their performance. We obtained similar results for the quadratic and MNIST databases

Challenges of implementation

- Needed to detach gradients from computational graph of pytorch to feed them to the meta optimizer
- Used CUDA to speed up the processing of the algorithm
- Had to make a new optimizee with the new parameters each time the meta optimizer is unrolled.
- Preprocess the gradients as the range can be quite large

$$\nabla^k \rightarrow \begin{cases} \left(\frac{\log(|\nabla|)}{p}, \operatorname{sgn}(\nabla)\right) & \text{if } |\nabla| \geq e^{-p} \\ (-1, e^p \nabla) & \text{otherwise} \end{cases}$$

Quadratic Loss function

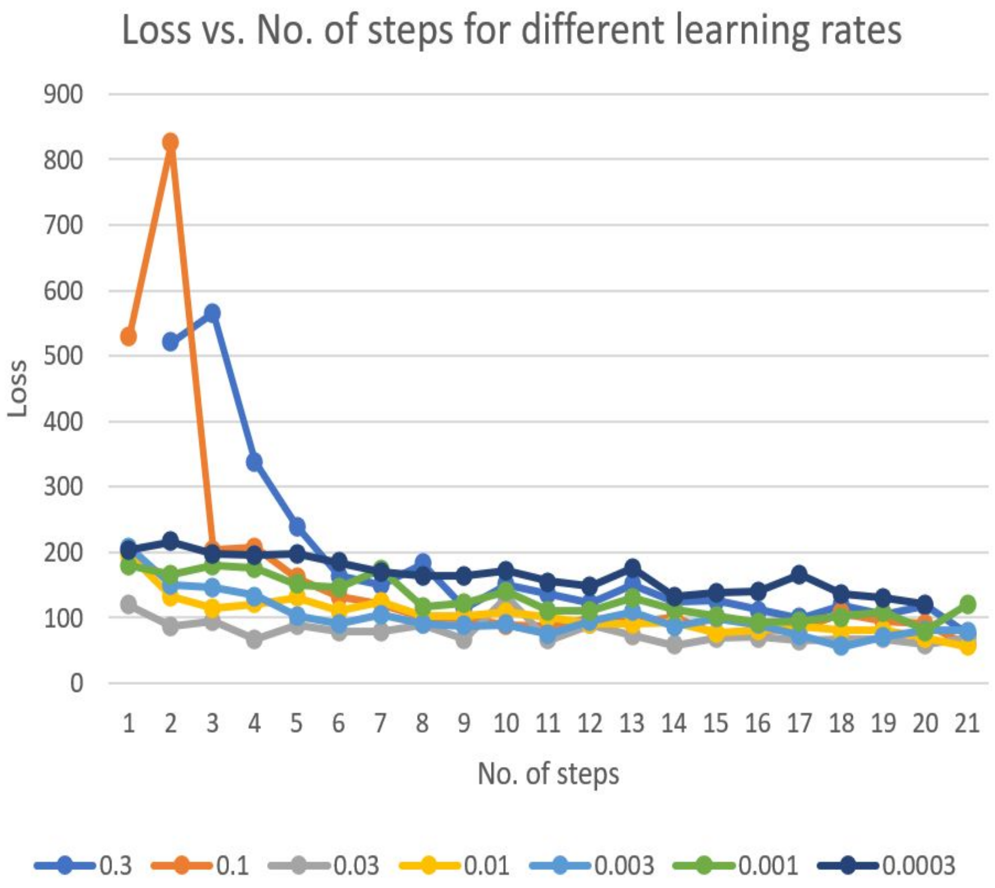
- Loss is defined simply as the quadratic loss:

$$L(x) = ||Wx - y||_2^2$$

- Objective: To find 10 dimensional vector x as close as possible to a given 10 dimensional vector y
- W is a 10*10 matrix

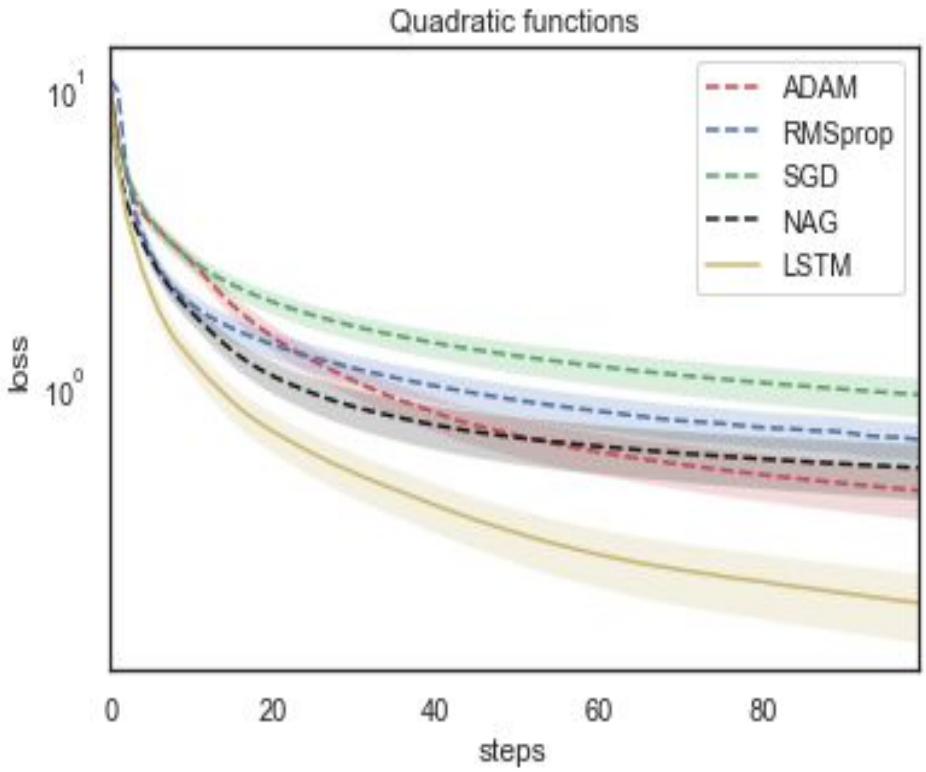
Best learning rate:

(For meta-optimiser)



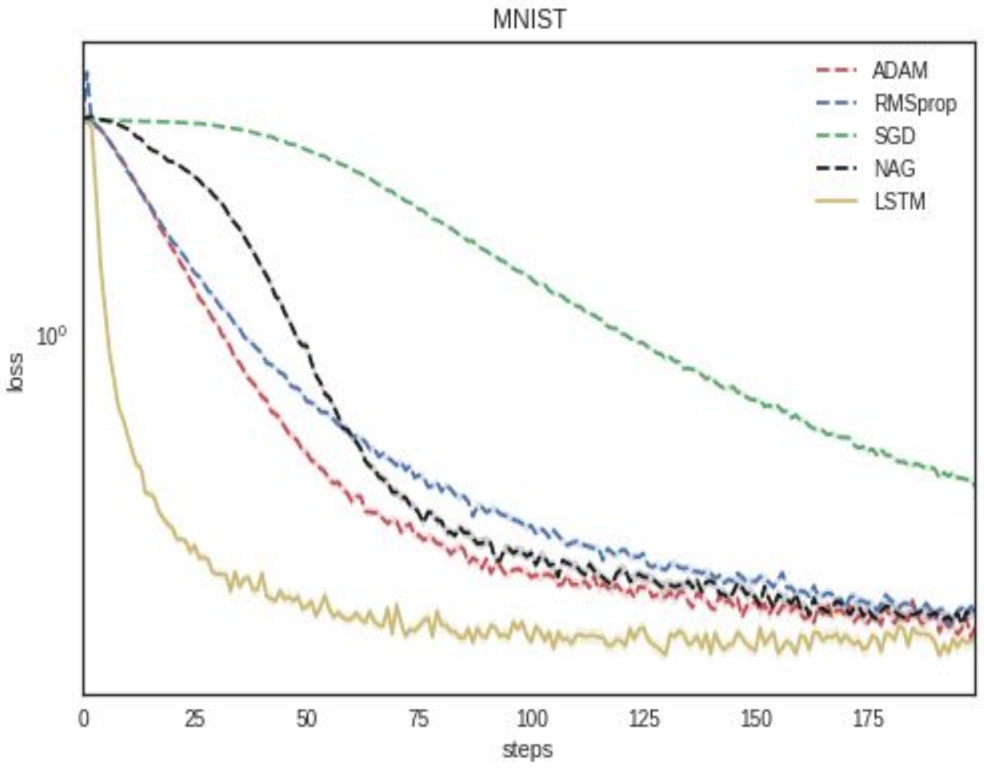
Hence the best learning rate= 0.003

Quadratic function: Comparison with hand-designed techniques



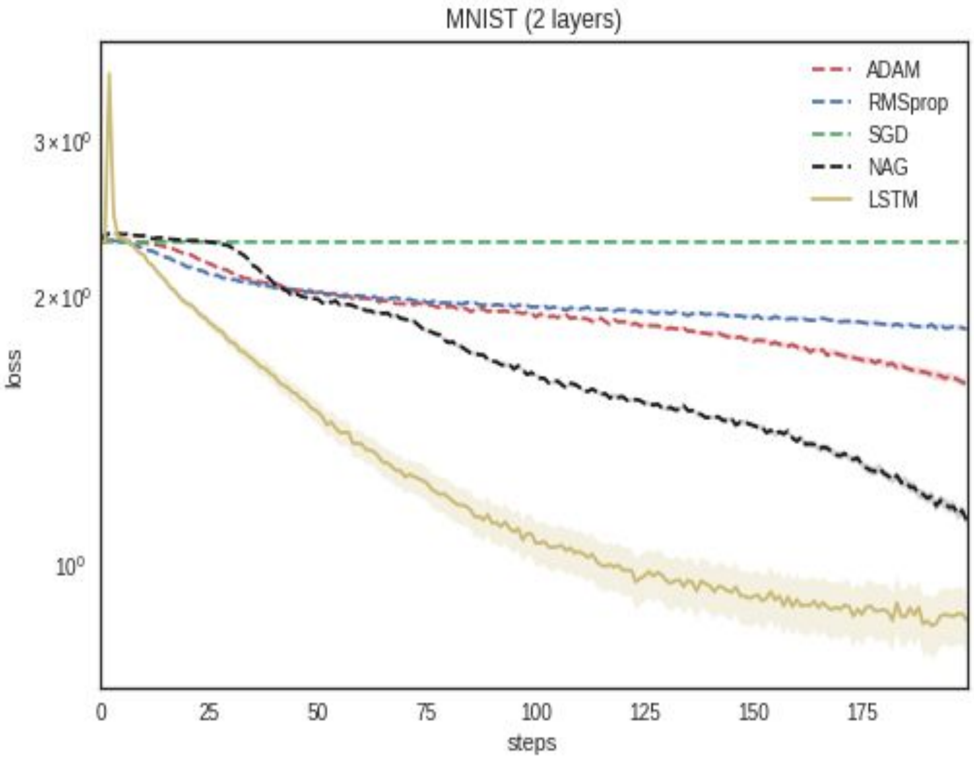
(W and y chosen randomly)

MNIST database: Comparison

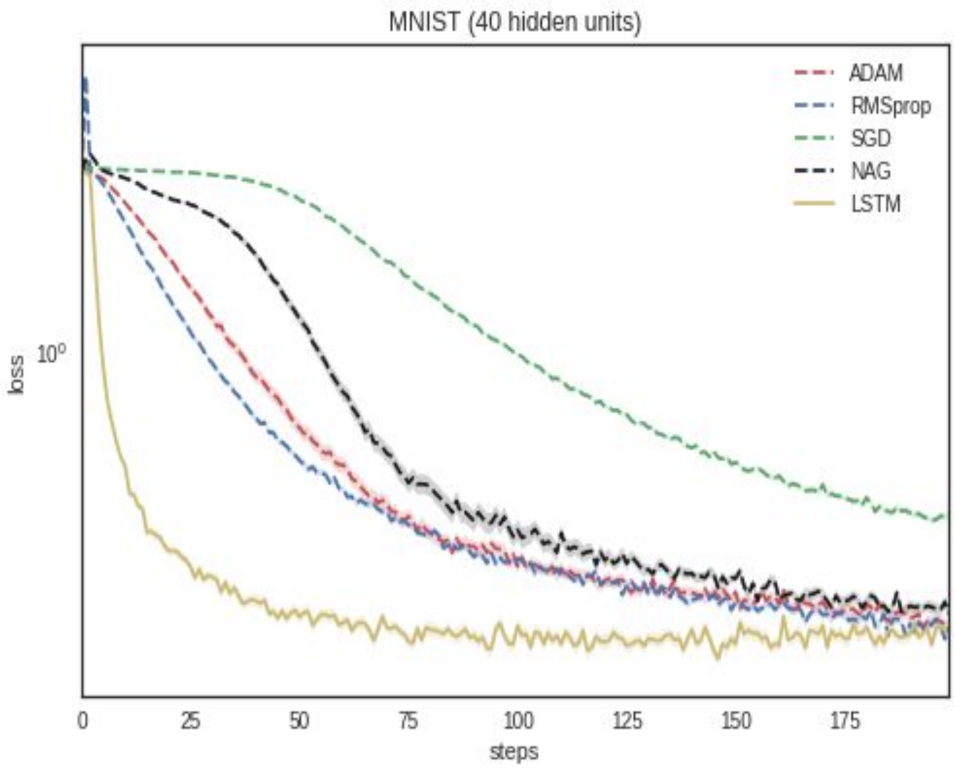


Sigmoid Function, 1 Hidden layer, 20 nodes

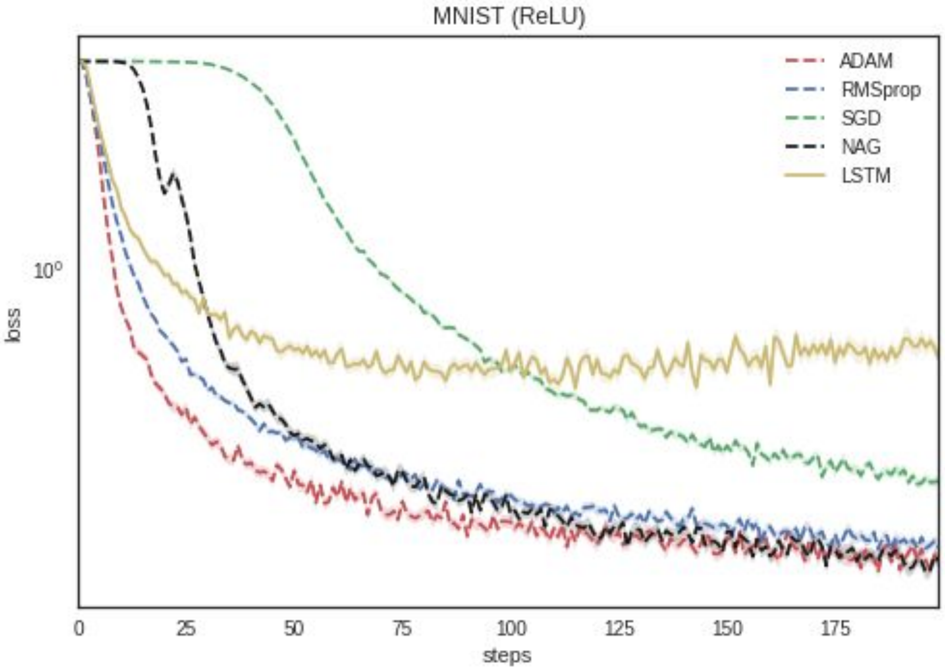
Variation:
2 Hidden Layers:



Variation: 40 hidden units



Variation: ReLU Activation Function



This is the only scenario where our LSTM meta-optimiser does not perform better than other standard methods

Proposal

- A three layer network in which the third network is used to decide the update rule for the optimiser which, in turns decides the update rule for the meta optimiser.
- The second network then will be a simple multi-perceptron network, and the third will be the LSTM cell used here.
- Since the challenge of programming just the meta optimizer is complicated enough for us, we have not tried to implement the structure here, but hope to work further.

Conclusion

- Meta-optimizer outperforms hand-designed algorithms in most of the cases
- But, this method is time-expensive so
  - It may not find use in live-time prediction algorithms (e.g. Automated driving)
  - But can certainly improve data analysis and prediction tasks (e.g. in fields of Astronomy and particle physics)
- Also emphasises the importance of what is called transfer learning where concepts learned in similar, simple settings can be used to predict outcomes in more complex scenarios.

References

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